

Numerical Analyses of CERN 200GeV/A Heavy-Ion Collisions Based on a Hydrodynamical Model with Phase Transition

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Abstract

We numerically analyze recent high energy heavy-ion collision experiments based on a hydrodynamical model with phase transition and discuss a systematic change of initial state of QGP-fluid depending on colliding-nuclei's mass.

In a previous paper, we formulated a (3+1)-dimensional hydrodynamical model for quark-gluon plasma with phase transition and discussed numerically the space-time evolution in detail. We here compare the numerical solution with the hadronic distributions given by CERN WA80 and NA35. Systematic analyses of the experiments with various colliding nuclei enable us to discuss the dependences of the initial parameters of the hydrodynamical model on colliding nuclei's mass. Furthermore, extrapolating the present experiments, we derive the possible hadronic distributions for lead-lead 150GeV/A collision.

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1 Introduction

Physics of quark-gluon plasma is one of the most important problems in high energy physics. Quark-gluon plasma state is expected to be produced in extremely high temperature, which will be realized in ultra-relativistic heavy ion collisions. To obtain the higher temperature, experimental setting has been growing in three directions: the higher incident energy per nucleon, the heavier incident nucleus, and the heavier target nucleus. In any case, colliding energy is getting larger, but the larger energy does not directly mean the higher temperature. It is not obvious, whether large fraction of colliding energy will be transferred into thermal energy (temperature) or kinetic energy (systematic flow). Especially, the heavier target does not correspond to the larger colliding velocity in the center of mass system. Hence, if we stand on the Landau-type “simple stopping” picture, achieved energy density must become lower for larger target mass. In this paper, we will analyze the recent experimental data for various target nuclei and discuss systematic change of initial parameters in hydrodynamical model.

In a previous paper[1] we formulated a semi-phenomenological quantum transport theory for a quark-gluon plasma fluid based on an operator valued Langevin equation. Putting a mode spectrum and a damping into this formula, we can easily evaluate thermodynamical quantities and transport coefficients. Introducing a simple model spectrum to this formula, we have already discussed the space-time evolution of (1+1)-dimensional viscous quark-gluon plasma fluid [2] and (3+1)-dimensional perfect fluid quark-gluon plasma with phase-transition [3]. We have also discussed the case of baryon-rich quark-gluon plasma with phase-transition [4], in which one of the conclusions was that the baryon-number effect was not so much large. Hence, although the existing experiments do not seem to be baryon-free, we

apply a simple baryon-free quark-gluon plasma model to analyses of available experiments.

2 Hydrodynamical model with phase-transition

Following [2], we here introduce a simple model mode spectrum,

$$\varepsilon(\mathbf{k}) = A\sqrt{\mathbf{k}^2 + M^2} \frac{1 - \tanh \frac{T-T_C}{d}}{2} + |\mathbf{k}| \frac{1 + \tanh \frac{T-T_C}{d}}{2} \quad (1)$$

where A is a parameter that adjusts the degrees of modes and T_C is the critical temperature responsible for the reliant phase transition. Supposing that the high temperature phase is dominated by massless u-, d-, s-quarks and gluons and the hadronic phase is dominated by pions and kaons, we can put $A = 1.89$, $M = 200$ MeV, $d = 2$ MeV, and $T_C = 160$ MeV. With these parameters, we can easily obtain an equation of state with phase transition-like behavior (fig.1), which seems to reproduce the Lattice QCD result [3].

The hydrodynamical equation is given by,

$$\partial_\mu T^{\mu\nu} = 0, \quad (2)$$

$$T^{\mu\nu} = EU^\mu U^\nu - P(g^{\mu\nu} - U^\mu U^\nu), \quad (3)$$

for perfect fluid. Here E, P and U^μ are, respectively, energy density, pressure and local four velocity. For cylindrically symmetric expansion along the collision axis, it is convenient to introduce new variables, τ, η, r and ϕ defined by

$$t = \tau \cosh \eta,$$

$$z = \tau \sinh \eta,$$

$$x = r \cos \phi,$$

$$y = r \sin \phi,$$

instead of ordinary coordinate t , x , y and z . As for the local four velocity, we represent it by four components, U^τ , U^η , U^r and U^ϕ associated with the new variables,

$$\begin{aligned} U^t &= U^\tau \cosh \eta + U^\eta \sinh \eta, \\ U^z &= U^\tau \sinh \eta + U^\eta \cosh \eta, \\ U^x &= U^r \cos \phi + U^\phi \sin \phi, \\ U^y &= U^r \sin \phi - U^\phi \cos \phi. \end{aligned}$$

Because of $U^\mu U_\mu = 1$ and the cylindrical symmetry, the four components can be reduced to two variables Y_T and Y_L [3],

$$\begin{aligned} U^\tau &= \cosh Y_T \cosh(Y_L - \eta), \\ U^\eta &= \cosh Y_T \sinh(Y_L - \eta), \\ U^r &= \sinh Y_T, \\ U^\phi &= 0. \end{aligned}$$

Taking account of the formation time, let us put the temperature distribution on η and r at $\tau = \tau_0$,

$$T(\tau_0, \eta) = T_0 \exp\left(-\frac{(|\eta| - \eta_0)^2}{3 \cdot 2 \cdot \sigma_\eta^2} \theta(|\eta| - \eta_0) - \frac{(r - r_0)^2}{3 \cdot 2 \cdot \sigma_r^2} \theta(r - r_0)\right), \quad (4)$$

assuming that the hydrodynamical expansion starts at $\tau = \tau_0 = 1$ fm later than the collision instance at $\tau = 0$ fm. T_0 represents the initial temperature, η_0 and r_0 are measures of the longitudinal and transverse spreads in η and r , respectively. T_0 , η_0 , r_0 , σ_η and σ_r are input parameters to characterize our model on which we should impose a constraint given by a fixed value of the initial fluid energy $E_{ini} = \chi E_{tot}$ (E_{tot} standing for the total collision energy and χ for the inelasticity). As for the initial condition of the local velocity, we use Bjorken's

scaling solution, $Y_L = \eta$, in the longitudinal direction, by which we can take the initial longitudinal flow into account. Initial values of these parameters should be determined on the basis of an appropriate physical discussion, but details of these parameters are deeply connected to the interaction mechanism and the thermalization process. In this paper, standing on *phenomenological* point of view, we try to choose these parameters so as to reproduce experimental results of both momentum spectrums.

3 Particle Distributions

The numerical solution of the hydrodynamical equation gives us the momentum distribution of hadrons, coming out from a local system with volume $d\sigma^\mu$ in local equilibrium with freeze-out temperature T_f , through the formula

$$\frac{d^3\Delta N}{d\mathbf{p}^3} = \frac{U^\mu p_\mu}{\sqrt{\mathbf{p}^2 + m^2}} \frac{1}{\exp(\frac{U^\rho p_\rho}{T_f}) - 1} \frac{U^\nu d\sigma_\nu}{(2\pi)^3} \quad (5)$$

where $p^0 = \sqrt{p^2 + m^2}$ with hadron mass m (see ref.[3]). Integrating eq.(5) on the hypersurface with T_f , we obtain

$$\frac{dN}{dY} = \int \frac{U^\mu p_\mu}{\exp(\frac{U^\rho p_\rho}{T_f}) - 1} \frac{U^\nu d\sigma_\nu P_T dP_T d\varphi}{(2\pi)^3} \quad (6)$$

for the Y-distribution,

$$\frac{dN}{d\eta'} = \int \frac{1}{\sqrt{\mathbf{p}^2 + m^2}} \frac{1}{\exp(\frac{U^\mu p_\mu}{T_f}) - 1} \frac{U^\nu d\sigma_\nu dP_L d\varphi}{(2\pi)^3}, \quad (7)$$

for η' -distribution, where $\eta' = \frac{1}{2} \ln \frac{P_- P_T}{P_+ P_L}$ stands for the pseudorapidity distribution, and

$$\frac{dN}{P_T dP_T} = \int \frac{U^\mu p_\mu}{\sqrt{\mathbf{p}^2 + m^2}} \frac{1}{\exp(\frac{U^\mu p_\mu}{T_f}) - 1} \frac{U^\nu d\sigma_\nu dP_L d\varphi}{(2\pi)^3} \quad (8)$$

for the P_T -distribution.

Our numerical results given by eq.(8) is shown in fig.2. As is well known, the freeze-out temperature T_f is directly connected with the P_T slope and we should determine it from the experimental P_T -spectrum. However, throughout this paper, we fix it equal to $T_f = 140\text{MeV}$ because of the lack of sufficient experimental data.

Our numerical solution of hydrodynamical model is so designed as to describe the head-on collision of the same kind of nuclei in the center of mass system. In order to apply the solution to the asymmetric collision, such as WA80 and NA35, we estimated effective participants in large target nucleus as,

$$A_{eff} = A_{tar} \left[1 - \left(1 - \left(\frac{A_{proj}}{A_{tar}} \right)^{\frac{2}{3}} \right)^{\frac{3}{2}} \right], \quad (9)$$

A_{tar} and A_{proj} being target mass and projectile mass, respectively.

4 Numerical results

Our hydrodynamical model contains many parameters to be adjusted. We have already fixed freeze-out temperature as $T_f = 140\text{MeV}$ from fig.2 in the previous section. We may put formation time equal to $\tau_0 = 1$ fm and initial transverse size equal to the size of smaller nucleus,

$$r_0 = 1.2 \times \sqrt[3]{A_{proj}} - \sigma_r \quad (\text{fm}), \quad (10)$$

where $\sigma_r = 1$ (fm). As for the local velocity, we use Bjorken's scaling solution, $Y_L = \eta$, neglecting initial transverse flow. Therefore, the residual parameters to be adjusted are only initial temperature T_0 at central point, and longitudinal size at initial time, η_0 and σ_η . We try to adjust these parameters so as to reproduce existing experiments. Our results for

sulphur beam experiments are shown in fig.3, and the values of parameters for sulphur and for oxygen beam are summarized in table 1. In any case, our model seems to reproduce the experimental results well with plausible values of the parameters. Table 2 stands for the results given by a equation of state in the limit $T_C \rightarrow \infty$, which corresponds to a hot hadron-gas model. In this case, χ became an unphysical value, larger than unity. It means that *we failed to reproduce experimental results with the hot hadron-gas model.*

These results tell us that larger nuclei are more effective to make a high temperature fluid than smaller nuclei can do even in target experiment. For larger targets, the initial temperature must become higher, the longitudinal size smaller, and the inelasticity larger. This kind of tendency is just desired for the experimental quark-gluon plasma production.

Sometimes, it has been pointed out that the present CERN energy is not high enough to make the Bjorken's scaling quark-gluon plasma, and the stopping picture of Landau type is then realized. However, the simple stopping picture is not in agreement with our results. In the simple stopping picture, the only collision parameter is the colliding relative velocity γ , and the longitudinal size contracts with $1/\gamma$ and the energy density increases with γ^2 . In target experiments, the larger target mass means the higher collision energy, but does not means the higher collision velocity. Table 3 shows the our results and relative velocities. The tendency is opposite to the simple Landau picture. Of course, it only means the failure of the *simple* stopping picture and we may describe the phenomena which require to introduce additional phenomenological parameters, such as energy dependent inelasticity [9].

In this paper, we have fitted hydrodynamical parameters in a purely phenomenological manner without resorting to any dynamical model for initial collision process. Although we need a dynamical model for the quark-gluon plasma formation to understand the results in

the table 1, we may extrapolate our phenomenological results in order to obtain the rough sketch of coming experiments. Figure 4 indicates us that parameters in Pb + Pb 150 GeV/A collision are $T_0 = 250\text{MeV}$ and $\eta_0 + \sigma_\eta = 0.8$. The possible pseudorapidity-distribution of hadrons is shown in fig. 5 and the total charged hadronic multiplicity (pions and kaons) reaches 2095.

5 Concluding Remarks

We have applied our (3+1)-dimensional hydrodynamical model with phase-transition to the recent heavy-ion experiments. Our results show that the inelasticity increases with target mass, the longitudinal size is getting smaller with target mass, and the initial temperature becomes higher. Such a tendency is very promising for our aim of producing quark-gluon plasma experimentally and also for understanding the future experimental results at CERN and RHIC.

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References

- [1] M. Mizutani, S. Muroya and M. Namiki, Phys. Rev. **D37** (1988), 3033.
- [2] Y. Akase, S. Daté, M. Mizutani, S. Muroya M. Namiki and M. Yasuda, Prog. Theor. Phys. **82** (1989), 591.
- [3] Y. Akase, M. Mizutani, S. Muroya and M. Yasuda, Prog. Theor. Phys. **85** (1991), 305. 1990-16.
- [4] T. Ishii and S. Muroya, Phys. Rev. **D46** (1992), 5156 .
- [5] R. Albrecht *et al.* :WA80, Z. Phys. **C 55** (1992), 539.
- [6] A. Bamberger *et al.* :NA35, Nucl. Phys. **A498** (1989), 133C.
- [7] T. Åkkeson *et al.* :HELIOS, Nucl. Phys. **B342** (1990), 279.
- [8] R. Albrecht *et al.* :WA80, Phys. lett. **B202** (1988), 596
- [9] U. Ornic, F. Pottag and R. M. Weiner Phys. Rev. Lett. **63** (1989),2641.

Table 1. Phase-Transition Model

Collision	A	σ_η	η_0	$T_0(\text{MeV})$	χ
S + Al ¹⁾	27 [†]	1.5	1.0	169	0.52
S + S ²⁾	32	1.5	1.0	167	0.46
S + Cu ¹⁾	64	1.5	0.8	180	0.69
S + Ag ¹⁾	107	1.5	0.7	188	0.75
S + W ³⁾	184	1.2	0.5	202	0.68
S + Au ¹⁾	197	1.5	0.7	195	0.88
O + Cu ⁴⁾	64	167	3.0	0.3	0.38
O + Ag ⁴⁾	107	172	3.0	0.2	0.54
O + Au ⁴⁾	197	176	3.0	0.1	0.62

Data 1) from [5], 2) from [6], 3) from [7] and 4) from [8]

Table 2. Hadron Model

Target	A	σ_η	η_0	$T_0(\text{MeV})$	χ
Al ¹⁾	27 [†]	0.8	0.5	335	0.82
S ²⁾	32	0.6	1.0	312	0.93
Cu ¹⁾	64	1.0	0.5	363	1.37
Ag ¹⁾	107	1.0	0.5	380	1.49
W ³⁾	184	0.8	0.7	405	1.65
Au ¹⁾	197	0.9	0.7	400	1.77

Data 1) from [5], 2) from [6] and 3) from [7]

Table 3. Parameters for Landau Picture

Collision	Effective Participants	E_{lab} (GeV)	E_{CM} (GeV)	γ_{cm}	$\sigma_\eta + \eta_0$
S + Al	30.9 + 27	6175.72	561.218	11	1.5 + 1.0
S + Cu	32 + 49.6	6400	774.461	8.3	1.5 + 0.8
S + Ag	32 + 63.0	6400	873.332	7.4	1.5 + 0.7
S + Au	32 + 81.1	6400	990.954	6.5	1.5 + 0.7
O + Cu	16 + 34.0	3200	453.783	7.1	3.0 + 0.3
O + Ag	16 + 41.9	3200	503.616	6.4	3.0 + 0.2
O + Au	16 + 52.7	3200	565.62	5.7	3.0 + 0.1

Figure Caption

Fig.1

The energy density as a function of temperature; the solid line stands for our phase-transition model, the dotted line for the simple hadron-gas of pions and kaons, and the dot-dashed line for the quark-gluon plasma of massless u-, d-, s-quarks and gluons. In the phase-transition model, phase-transition takes place at $T_C = 160\text{MeV}$ in the width of $d = 2\text{MeV}$.

Fig.2

The P_T spectrum of pions in S+S 200 GeV/A collision together with the data of NA35. The solid line stands for $T_f = 140\text{ MeV}$, the dashed line for $T_f = 120\text{MeV}$, the dotted line for $T_f = 100\text{MeV}$, and dot-dashed line for $T_f = 80\text{MeV}$, respectively.

Fig.3

The pseudorapidity distributions of charged hadron in 200GeV/A Sulphur beam collision. The solid line stands for S+Al, the dashed line for S+Cu, the dotted line for S+Ag, and the dot-dashed line stands for S+Au, respectively. In our calculation, we take account of pions and kaons only. Plots are the data from [5]. In the case of S+Al, r_0 is determined from the size of Al.

Fig.4

The values of initial parameters as a function of Total Collision Energy. The solid line stands for initial temperature, T_0 , and the dashed line for longitudinal extent, $\eta_0 + \sigma_\eta$. The

plots from the left to the right correspond to O+Cu, O+Ag, O+Au, S+Al, S+Cu, S+Ag, and S+Au, respectively. Plots at the right end are the extrapolation of parameters to Pb+Pb 150 GeV/A collision.

Fig.5

The psuedorapidity distribution of charged hadrons (π^\pm and K^\pm) in Pb+Pb 150 GeV/A collision. Here, we put $T_0=250\text{MeV}$, $\eta_0=0.2$, and $\sigma_\eta=0.6$.

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